

**R18**

Code No: 154BG

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year II Semester Examinations, September/October - 2023

LAPLACE TRANSFORMS, NUMERICAL METHODS AND COMPLEX VARIABLES

(Common to EEE, ECE, EIE)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART - A**

**(25 Marks)**

1.a) Find the inverse the Laplace transforms of  $\frac{1}{(s-2)(s-3)}$ . [2]

b) Find the Laplace transforms of  $(1+te^{-t})^3$ . [3]

c) Write the Bisection method formula to solve an equation  $f(x) = 0$ . [2]

d) Show that  $(1+\Delta)(1-\nabla) \equiv 1$ . [3]

e) Write the formula for Simpson's 1/3 rule for numerical integration of the function  $f(x)$  in the interval  $[x_0, x_n]$ . [2]

f) Write the formula for modified Euler's method to find  $f(x_0 + h)$  from the initial value

problem  $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ . [3]

g) Compute the limit  $\lim_{z \rightarrow 1} \frac{z^3 - 1}{z - 1}$ . [2]

h) Separate the real and imaginary parts of the function  $f(z) = z^2 - z + 2i$ . [3]

i) Define Maximum-Modulus theorem. [2]

j) Define the zeros of a function. Give an example. [3]

**PART - B**

**(50 Marks)**

2.a) Find the Laplace transform of the "triangular" wave function  $f(t)$ , defined as

$$f(t) = \begin{cases} t, & \text{in } 0 \leq t \leq a \\ 2a - t, & \text{in } a \leq t \leq 2a \end{cases} \quad f(t+2a) = f(t)$$

b) Find the inverse Laplace transform of the function  $\log \frac{s^2 + 1}{s(s+1)}$ . [5+5]

**OR**

3.a) Find inverse Laplace transform of the function  $\frac{s^3}{(s^2+a^2)^2}$  using convolution theorem.

b) Solve the following differential equation, using Laplace transform. [5+5]  
 $y'' - 3y' - 4y = 2e^{-t}$ ,  $y(0) = y'(0) = 1$ .

4.a) Estimate the missing terms in the following table:

$x$	0	1	2	3	4
$y=f(x)$	1	3	9	?	81

b) Obtain the value of  $t$  when  $A = 85$  from the following table using Lagrange's method.

$t$	2	5	8	14
$A$	94.8	87.9	81.3	68.7

[5+5]

**OR**

5.a) From the following table find the number of students who obtained less than 46 marks using interpolation.

Marks	No. of students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

b) Find a real root of the equation  $x^3 - 5x + 1 = 0$ , using Regula- falsi method correct to three decimal places. [5+5]

6.a) Evaluate  $\int_0^9 \frac{dx}{1+x^3}$  correct to four decimal places by Simpson's 3/8<sup>th</sup> rule.

b) Given that  $dy/dx = x^2 + y$  and  $y(0) = 1$ . Find an approximate value of  $y(0.1)$ , taking  $h = 0.05$  by the modified Euler's method. [5+5]

**OR**

7.a) Using the Runge-Kutta method of order 4, find  $y(0.2)$  given that  $dy/dx = 3x + y^2$ ,  $y(0) = 1$  taking  $h = 0.1$ .

b) Find the value of  $\log 2$  from  $\int_0^1 \frac{x^2}{0.1+x^3} dx$ , using Simpson's 1/3<sup>rd</sup> rule, by dividing the range into four equal parts. [5+5]

8.a) If  $u(r, \theta) = \left(r + \frac{1}{r}\right) \sin \theta$ , ( $r \neq 0$ ), find its harmonic conjugate.

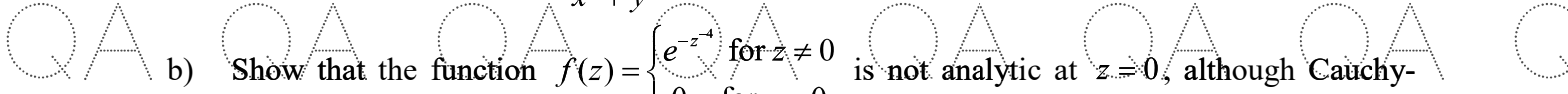
b) Find an analytic function  $f(z) = u(x, y) + iv(x, y)$ , where  $u(x, y) = x^3 - 2xy - 3xy^2$ , using Milne-Thomson method. [5+5]

**OR**

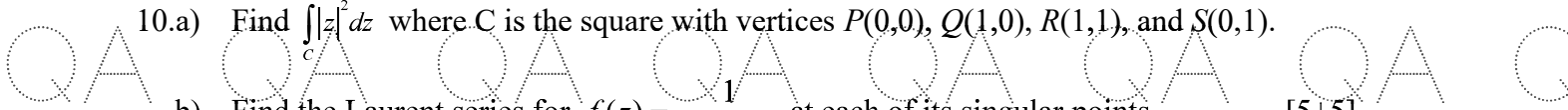


9.a) Compute the Laplacian, where it exists for the following functions:

- i)  $\log(xy)$  and ii)  $\frac{x^2 - y^2}{x^2 + y^2}$ .



b) Show that the function  $f(z) = \begin{cases} e^{-z^{-4}} & \text{for } z \neq 0 \\ 0 & \text{for } z = 0 \end{cases}$  is not analytic at  $z=0$ , although Cauchy-Riemann equations satisfy at that point. How would you explain this? [5+5]



10.a) Find  $\int_C |z|^2 dz$  where  $C$  is the square with vertices  $P(0,0)$ ,  $Q(1,0)$ ,  $R(1,1)$ , and  $S(0,1)$ .

b) Find the Laurent series for  $f(z) = \frac{1}{z^2(z-1)}$  at each of its singular points. [5+5]

**OR**

11.a) Compute the integral  $\int_{|z|=2} f(z) dz$ , where  $f(z) = \frac{e^z}{(z-1)^2(z-i)(z-3)}$  using Residue theorem.



b) Use contour integration to evaluate the integral  $\int_0^{2\pi} \frac{d\theta}{\cos \theta + i\sqrt{3}}$ . [5+5]

---ooOoo---

